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$\bar{\mu}$  - MESIC MOLECULAR IONS AND  
 $\bar{\mu}$  - MESIC MOLECULAR PROCESSES IN HYDROGEN

(*вопросы*)  
установления

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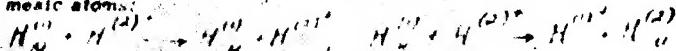
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*M* - MESIC MOLECULAR IONS AND  
*M* - MESIC MOLECULAR PROCESSES IN HYDROGEN

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## ABSTRACT

A number of atomic and mesic molecular processes in the system of hydrogen isotopes are considered (molecular ion formation  
 $H_N + H^{(1)} \rightarrow (H^{(1)} H^{(1)})_N$ , elastic collisions and charge exchange  
of mesic atoms).

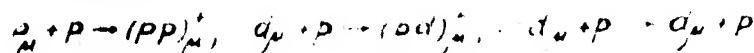


Energy levels of mesic molecules are calculated. The numerical calculations were carried out on BEAM computer. Corrections due to the motion of nuclei up to the first order with respect to  $\frac{M}{m}$  are taken into account.

A special property of mesic hydrogen atoms is their neutrality since at distances larger compared to the radius of Bohr mesic atom orbit ( $2.5 \cdot 10^{-11}$  cm) the nucleus charge is almost completely screened by the meson charge. This circumstance provides a number of mesic molecular processes in hydrogen (or in the mixture of hydrogen isotopes) such as  $\mu^-$ -meson exchange between various nuclei (charge exchange), formation of mesic molecules etc. These processes define to a large extent the catalysis of nuclear reactions in hydrogen predicted by Frank<sup>1</sup>, Zeldovich<sup>2</sup> and Sacharov<sup>3</sup> and investigated experimentally in papers 4-5. On the other hand as it was pointed out in<sup>6-8</sup>, mesic molecular processes in hydrogen are important in the experimental determination of the law of interaction  $\mu^- + p \rightarrow \alpha + \gamma$  (in particular, in order to distinguish experimentally  $V = s$  and  $V = A$  forms<sup>3</sup>).

Some mesic molecular processes in hydrogen have been studied in earlier papers.

In the paper<sup>11</sup> the levels of mesic molecular ions  $(pp)_\mu^+$ ;  $(pd)_\mu^+$ ;  $(dd)_\mu^+$ ; and cross sections of the processes



have been calculated. There is a great discrepancy with our data for the upper level of  $(dd)_\mu^+$  with  $L=0$ . Our cross section of the scattering of  $\mu^-$  by protons decreases in volume with energy decrease.

In<sup>12</sup> the probability of the charge exchange  $\mu^- + \alpha \rightarrow \mu^- + p$  has been calculated by the method similar to that used in the present paper. However, for the potentials  $f_{gg}$  and  $f_{\alpha\alpha}$  and for the corrections  $K_{gg}$  and  $K_{\alpha\alpha}$  were used functions than those given in<sup>4,15</sup> has been taken. Moreover, the authors<sup>12</sup> have expected that for  $R \geq 6$  the exact solutions of the system coincide with their asymptotic values, what is not quite correct.

In paper<sup>18</sup> the ground state levels in mesic molecular ions have been calculated. The corrections to the potential energy of  $\mu^- + \frac{3}{2}H$  have been neglected. Besides, this in order to find the eigenvalues for mesic molecules with different nuclei we need to solve a system of two equations (owing to the presence of the nuclear moments which provide transitions between the states  $H_N$  and  $\frac{3}{2}H$ ).

In<sup>20</sup> the levels in mesic molecular ions  $(pp)_\mu^+$  and  $(pd)_\mu^+$  are fairly well determined (the

corrections due to the motion of nuclei are small. The equation for the wave function is less accurate than in 14.

In paper<sup>14</sup> the estimates of basic levels for molecular ions and interactions  $d + p \rightarrow d + p, d + p \rightarrow pd$  are given.

Let us consider a system involving the two nucleons and hydrogen isotopes with masses  $M_1$  and  $M_2$  and the  $\mu^-$ -meson. Let  $\vec{z}_1, \vec{R}_1, \vec{R}_2$  be the coordinates of  $\mu^-$ -meson and nuclei\*. The Hamiltonian of the nuclear-splinter interactions are neglected is of the form:

$$\hat{H} = -\frac{1}{2} \Delta_{\mu} - \frac{1}{2M_1} \Delta_{R_1} - \frac{1}{2M_2} \Delta_{R_2} - \frac{1}{r_1} - \frac{1}{r_2} + \frac{1}{R} \quad (1)$$

where

$$r_1 = |\vec{z}_1 - \vec{R}_1|; \quad r_2 = |\vec{z}_2 - \vec{R}_2|, \quad R = |\vec{R}_2 - \vec{R}_1|$$

Believing that the  $\mu^-$ -meson is on the  $K^-$ -orbit we shall seek for the wave function of the system in the form:

$$\psi = \psi(\vec{R}) \Sigma_g(R, \vec{z}) + H(\vec{R}) \Sigma_n(R, \vec{z}) \quad (2)$$

where  $\psi(\vec{R})$  and  $H(\vec{R})$  describe the relative motion of nuclei, and  $\Sigma_g$  and  $\Sigma_n$  represent the wave functions of  $\mu^-$ -meson in the field of fixed nuclei which are from one other at the distance  $R$  \*\*. When  $R \rightarrow \infty$

$$\Sigma_g \rightarrow \frac{1}{\sqrt{2\pi}} (e^{-r_1} + e^{-r_2}), \quad \Sigma_n \rightarrow \frac{1}{\sqrt{2\pi}} (e^{-r_1} - e^{-r_2}) \quad (3)$$

and when  $R \rightarrow 0$   $\Sigma_g$  and  $\Sigma_n$  transform into the wave functions of the  $H_e^-$  ion states  $1S$  and  $2P$  correspondingly.

Substituting (2) into the Schrödinger equation

$$\hat{H}\psi = E\psi \quad (4)$$

and having in mind that the wave functions  $\Sigma_g$  and  $\Sigma_n$  satisfy the equation:

$$\left( -\frac{1}{2} \Delta_{\vec{z}} - \frac{1}{r_1} - \frac{1}{r_2} + \frac{1}{R} \right) \Sigma_i = E_i(R) \Sigma_i(\vec{z}, R) \quad (5)$$

we shall obtain after multiplying by  $\Sigma_g$  and  $\Sigma_n$  and integrating over the  $\mu^-$ -meson coordinates the system of equations for  $\psi(\vec{R})$  and  $H(\vec{R})$ :

\* Suppose we shall use the muon atom units  $\hbar = 1, e = 1, m_\mu = 207m_e = 1$  the muon atom unit of length is  $1/m_\mu \ell = 2.56 \cdot 10^{-11} \text{ cm}$  while the present units of energy  $m_\mu \ell n / \hbar^2 = 5.6 \cdot 10^3 \text{ ev} 1$ .

\*\* Let us note that since the wave function (2) depends only on the differences of particle coordinates in the center-of-mass system the chosen coordinate system is at rest.

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$$-\frac{1}{2M_{12}} \Delta \vec{R} \Psi + \left( E_g + \frac{1}{2M_{12}} K_{gg} \right) \Psi + \frac{1}{2M_{12}} K_{gu} H - \frac{1}{M_{12}} Q_{gu} \nabla_R H = E \Psi \quad (6)$$

$$-\frac{1}{2M_{12}} \Delta \vec{R} H + \left( E_u + \frac{1}{2M_{12}} K_{uu} \right) H + \frac{1}{2M_{12}} K_{ug} \Psi - \frac{1}{M_{12}} Q_{ug} \nabla_R \Psi = E H$$

where  $\frac{1}{M_{12}} = \frac{1}{M_1} + \frac{1}{M_2}$ ;  $\vec{R} = \vec{R}_1 - \vec{R}_2$ , and the functions  $\frac{1}{2M_{12}} K_{ig}$ ;  $\frac{1}{M_{12}} Q_{ig}$  are the matrix elements between the functions  $\Sigma_g$  and  $\Sigma_u$  of operators

$$\frac{1}{2M_{12}} \hat{K} = -\frac{1}{2} \left( \frac{1}{M_1} \Delta \vec{R}_1 + \frac{1}{M_2} \Delta \vec{R}_2 \right) \quad (7)$$

$$\frac{1}{M_{12}} \hat{Q} = \left( -\frac{1}{M_1} \nabla \vec{R}_1 + \frac{1}{M_2} \nabla \vec{R}_2 \right) \quad (8)$$

It is easy to show that due to the normalization conditions of  $\Sigma_g$  and  $\Sigma_u$  the matrix elements of the operator  $\hat{Q}$  are equal zero while the non-diagonal ones are opposite in sign in view of the orthogonality of  $\Sigma_g$  and  $\Sigma_u$ .

$$Q_{gu} = -Q_{ug} = Q \quad (9)$$

If we use the property of symmetry of  $\Sigma_g$  and  $\Sigma_u$  with respect to the exchange of nuclei we can separate the dependence on masses in matrix elements

$$K_{ii} = \int \Sigma_i (-\Delta \vec{R}_i) \Sigma_i d\tau \quad i = g, u \quad (10)$$

$$K_{ij} = \frac{M_2 - M_1}{M_2 + M_1} \int \Sigma_i (-\Delta \vec{R}_i) \Sigma_j d\tau \quad (11)$$

$$\tilde{Q} \cdot \frac{M_2 - M_1}{M_2 + M_1} \int \Sigma_g (-\nabla \vec{R}) \Sigma_u d\tau = Q \cdot \frac{\vec{R}}{R} \quad (12)$$

Thus, if the nuclei are identical  $M_1 = M_2$  the system of equations (6) breaks up into two independent equations. This result is quite clear since the wave function (2) for identical nuclei in virtue of the symmetry can include only one term (either  $\Sigma_g$  or  $\Sigma_u$ ). The terms  $\frac{1}{2M_{12}} K_{ij}(R)$ ,  $\frac{1}{M_{12}} Q(R)$  represent themselves the corrections to the adiabatic potentials due to the motion of nuclei (with the accuracy up to the first order with respect to  $R/M_{12}$ ). Since the ratio  $R/M_{12}$  for the meson is not so small as for the electron the above terms contribute

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essentially. For  $R \rightarrow \infty$  the value  $Q \rightarrow 0$  and the terms  $\frac{1}{2M_{12}} K_{ij}(\infty)$  represent the corrections which take into account the reduced masses of the separated mesic atoms with the accuracy  $(\frac{m_u}{M_1})^2, (\frac{m_u}{M_2})^2$ . Noting that the energy of separated mesic atoms (in mesic atom units) is

$$\begin{aligned} E_1^0 &= -\frac{1}{2} \frac{M_1}{M_1+1} \approx -\frac{1}{2} + \frac{1}{2M_1}, \\ E_2^0 &= -\frac{1}{2} \frac{M_2}{M_2+1} \approx -\frac{1}{2} + \frac{1}{2M_2}, \end{aligned} \quad (13)$$

and taking into account that the matrix elements of the operator  $(-\Delta \vec{R})$  for  $R \rightarrow \infty$  equal to (Appendix) we can write:

$$\left\{ E_g(\infty) + \frac{1}{2M_{12}} K_{gg}(\infty) \right\} \left\{ E_u(\infty) + \frac{1}{2M_{12}} K_{uu}(\infty) \right\} = \frac{1}{2} (E_1^0 + E_2^0) \quad (14)$$

$$\frac{1}{2} K_{gu}(\infty) = \frac{1}{2M_{12}} K_{gu}(\infty) = \frac{1}{2} (E_1^0 - E_2^0) = \frac{1}{2} \Delta E \quad (15)$$

Separating in the quantities  $E_g(R), E_u(R)$  and  $\frac{1}{2M_{12}} K_{ij}(R)$  their values for  $R = \infty$

$$\begin{aligned} E_{g,u}(R) &= E_{g,u}(\infty) + E'_{g,u}(R) \\ K_{ij}(R) &= K_{ij}(\infty) + K'_{ij}(R) \end{aligned} \quad (16)$$

we rewrite the system of equations (6) in the form

$$\begin{aligned} -\frac{1}{2M_{12}} \Delta R \Psi + (E' + \frac{1}{2M_{12}} K_{gg'}) \Psi + (\frac{1}{2} \Delta E + \frac{1}{2M_{12}} K'_{gu}) H - \frac{1}{M_{12}} Q \frac{dH}{dR} &= \\ -\frac{1}{2M_{12}} \Delta R H + (E'_u + \frac{1}{2M_{12}} K_{uu'}) H + (\frac{1}{2} \Delta E + \frac{1}{2M_{12}} K'_{ug}) \Psi + \frac{1}{M_{12}} Q \frac{d\Psi}{dR} &= E'H \end{aligned} \quad (6a)$$

where  $\Delta E = E_1^0 - E_2^0$  and the energy  $E'$  is calculated from the middle between the levels of separated mesic atoms:

$$E' = \frac{1}{2} (E_1^0 + E_2^0) \quad (17)$$

Since the nuclei are identical  $E'$  is calculated simply from the level of separate mesic atom.  
Separating the angular dependence  $\Psi(\vec{R})$  and  $H(\vec{R})$

$$g_i(\vec{R}) = \frac{1}{R} g_i(R) Y_{L,M_L}(\theta, \varphi)$$

$$h_i(\vec{R}) = \frac{1}{R} h_i(R) Y_{L,M_L}(\theta, \varphi) \quad (18)$$

where  $Y_{L,M_L}(\theta, \varphi)$  is the spherical function we obtain for  $g(R)$  and for  $h(R)$  the equations:

$$-\frac{1}{2M_2} \frac{d^2 g_i}{dR^2} + \left( E_g + \frac{1}{2M_2} K'_{gg} + \frac{L(L+1)}{2M_2 R^2} \right) g_i + \left( \frac{1}{2} \Delta E + \frac{1}{2M_2} K'_{gg} \right) h_i - \frac{1}{M_2} Q \cdot R \frac{d}{dR} \left( \frac{h_i}{R} \right) = E_g g_i$$

$$-\frac{1}{2M_2} \frac{d^2 h_i}{dR^2} + \left( E_u + \frac{1}{2M_2} K'_{uu} + \frac{L(L+1)}{2M_2 R^2} \right) h_i + \left( \frac{1}{2} \Delta E + \frac{1}{2M_2} K'_{uu} \right) g_i + \frac{1}{M_2} Q \cdot R \frac{d}{dR} \left( \frac{g_i}{R} \right) = E_u h_i \quad (19)$$

The potentials  $E_g(R)$  and  $E_u(R)$  were determined by solving the Eqs. (5) by many authors, starting with the paper [17]. In the present paper the values  $E_g(R)$  and  $E_u(R)$  taken from [14] have been used. The values of the functions  $K'_{gg}(R)$  and  $K'_{uu}(R)$  may be got by means of the recalculation from [15] and the values  $Q(R), K'_{ug}(R), K'_{gu}(R)$  are calculated in the approximation of 'united atom' (UA) and 'linear combination of atomic orbits' (LCAO) respectively for small and large values of  $R$  (for details see Appendix). For investigating the asymptotic behaviour of the solution for  $R \rightarrow \infty$  it is convenient to introduce the functions

$$u_i(R) = \{g_i(R) + h_i(R)\}/\sqrt{2}; \quad b_i(R) = \{g_i(R) - h_i(R)\}/\sqrt{2} \quad (20)$$

Comparing (2) and (3) it is easy to see that the functions  $u(R)$  and  $b(R)$  for  $R \rightarrow \infty$  describe a radial motion of the nucleus with mass  $M_2$  with respect to mesic atom  $M_1$  and of the nucleus with mass  $M_1$  with respect to mesic atom with mass respectively  $M_2$ .

The functions (20) satisfy the equations:

$$-\frac{1}{2M_2} \frac{d^2 u_i}{dR^2} + \left\{ \frac{1}{2} E_g + \frac{1}{2M_2} K'_{gg} + \left( E_u + \frac{1}{2M_2} K'_{uu} \right) \right\} +$$

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$$\begin{aligned}
 & -\frac{1}{2M_{12}} (K'_{gu} + K'_{ug}) + \frac{\Delta E}{2M_{12} R^2} + \frac{\Delta E}{2} \} a_L \cdot \left\{ \frac{1}{2} \left[ \left( L_g \cdot \frac{1}{2M_{12}} K'_{gg} \right) - \left( E_u \cdot \frac{1}{2M_{12}} K'_{uu} \right) \right] - \right. \\
 & \left. - \frac{1}{2M_{12}} (K'_{gu} - K'_{ug}) \right\} b_L \cdot \frac{QR}{M_{12}} \frac{d}{dR} \left( \frac{f}{R} \right) = E' a_L \\
 & \frac{d^2 a_L}{dR^2} \cdot \left\{ \frac{1}{2} \left[ \left( E_g \cdot \frac{K'_{gg}}{2M_{12}} \right) + \left( E_u \cdot \frac{K'_{uu}}{2M_{12}} \right) \right] - \frac{1}{4M_{12}} (K'_{gu} - K'_{ug}) \cdot \frac{\Delta E}{2M_{12} R^2} - \frac{\Delta E}{2} \right\} b_L \cdot \\
 & \left[ \frac{1}{2} \left[ \left( E_g \cdot \frac{K'_{gg}}{2M_{12}} \right) - \left( E_u \cdot \frac{K'_{uu}}{2M_{12}} \right) \right] \cdot \frac{1}{4M_{12}} (K'_{gu} - K'_{ug}) \right] a_L \cdot \frac{QR}{M_{12}} \frac{d}{dR} \left( \frac{a_L}{R} \right) = E' b_L
 \end{aligned}$$

and the boundary condition for  $R = 0$

$$a(0) = b(0) = 0 \quad (22)$$

Eqs. (21) for  $R \rightarrow \infty$  are of the form:

$$\begin{aligned}
 & -\frac{1}{2M_{12}} \frac{d^2 a}{dR^2} = \left( E' - \frac{1}{2} \Delta E \right) \cdot a \\
 & -\frac{1}{2M_{12}} \cdot \frac{d^2 b}{dR^2} = \left( E' + \frac{1}{2} \Delta E \right) \cdot b
 \end{aligned} \quad (23)$$

Let  $M_1 \neq M_2$ , so that for definiteness  $\Delta E > 0$ . Then three types of motion are possible depending on the value of  $E'$ .

a) If  $E' > \frac{1}{2} \Delta E$ , i.e. the energy is higher than the  $K$ -level of lighter mesic atom. The meson may be near the nucleus  $M_1$  as well as near nucleus  $M_2$ . If at first the meson was near the nucleus  $M_1$  and there is charge exchange to the nucleus  $M_2$  then it is necessary that the wave functions would obey the following condition: that  $b(R)$  for  $R \rightarrow \infty$  must include only a divergent wave

$$u(R) \approx C_1 e^{iK_1 R} + C_2 e^{-iK_1 R} \quad (24)$$

$$\alpha(R) \approx C_3 e^{i\chi_2 R} + C_4 e^{-i\chi_2 R} \quad \text{where } C_4 = 0 \quad (25)$$

b)  $E' - \frac{1}{2} \Delta E < E < E' + \frac{1}{2} \Delta E$  i.e. the energy lies between separated mesic atoms. When  $R \rightarrow \infty$  the meson cannot be near the separated lighter nucleus because of lack of energy. This case corresponds to the scattering of mesic atom with nucleus  $M_2$  by the nucleus  $M_1$ , without the possibility of charge exchange. The wave functions should obey the condition in accordance to which the function  $\alpha(R)$  for  $R \rightarrow \infty$  should not contain an exponentially increasing term:

$$\begin{aligned} \alpha(R) &\approx D_1 e^{-x_1 R} + D_2 e^{x_1 R} \\ &D_2 = 0 \\ \delta(R) &\approx D_3 e^{-i\chi_2 R} + D_4 e^{i\chi_2 R} \end{aligned} \quad (26)$$

$$x_1^2 = -K_1^2 = -2M \left( E' - \frac{1}{2} \Delta E \right) \quad (27)$$

It is obvious that the conditions (22) and (26) (as well as the conditions (22) and (24)) can be satisfied at any energy, taken from the considered energy region and determine the solution of the system of equations (21) with the accuracy up to the normalization.

c)  $E' - \frac{1}{2} \Delta E$  is a region of the discrete spectrum corresponding to the bound states of mesic molecules. For  $R \rightarrow \infty$  two conditions are imposed to the solution of the system: the absence of the increasing exponents in both functions  $\alpha(R)$  and  $\delta(R)$

$$\begin{aligned} \alpha(R) &\approx Z_1 e^{-x_1 R} + T_2 e^{x_1 R} \\ \delta(R) &\approx Z_3 e^{-x_2 R} + T_4 e^{x_2 R} \end{aligned} \quad (28)$$

$$T_2(E') = 0, \quad T_4(E') = 0 \quad (29)$$

$$x_1^2 = 2M_{12} \left( |E'| + \frac{1}{2} \Delta E \right); \quad x_2^2 = 2M_{12} \left( |E'| - \frac{1}{2} \Delta E \right)$$

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These conditions (26) and (22) may be satisfied only for definite values of  $E'$ , being energy levels of the basic molecule. In order to find solutions satisfying conditions (22) for  $R=0$  and the asymptotic conditions of the type (24), (26) and (28) when  $R \rightarrow \infty$  we can find two linearly independent solutions satisfying the condition (22) and construct a linear combination satisfying the asymptotic conditions for  $R \rightarrow \infty$ . Note that between two any solutions of the system of equations of the second order (16) there exists a connection which can be easily established if we take into account that

$$\psi_{g_1} - K_{gg} \psi_g = 2dw\theta \quad (30)$$

Indeed, if  $\{\psi_g, H_1\}$  and  $\{\psi_{g_1}, H_2\}$  are the solutions of the system (16), then:

$$d_R \left( (\psi_{g_1} + \psi_g - \psi_g - \psi_{g_1}) \cdot (H_2 \oplus H_1 - H_1 \oplus H_2) + 2Q(H_1 \psi_{g_1} - H_2 \psi_g) \right) = 0 \quad (31)$$

For the functions (20), satisfying the condition (22) the identity (31) takes the form:

$$(a_2 \frac{da}{dR} - a_1 \frac{da_1}{dR}) + (b_2 \frac{db}{dR} - b_1 \frac{db_1}{dR}) + 2Q(a_1 b_2 - a_2 b_1) = 0 \quad (32)$$

The relation (32) may be used for testing the corrections of the numerical integration. As linearly independent solutions for the Eqs. (21) we may choose, for example, the solutions determined for  $R = 0^\circ$  by the conditions:

$$\begin{cases} a(0) = b(0) = 0 \\ a'(0) = b'(0) = 1 \end{cases} \quad \begin{cases} g = h = 0 \\ g' = \sqrt{2}; h' = 0 \end{cases} \quad (33)$$

$$\begin{cases} a(0) = b(0) = 0 \\ a'(0) = -b(0) = 1 \end{cases} \quad \begin{cases} g = h = 0 \\ g' = 0; h' = \sqrt{2} \end{cases} \quad (34)$$

### CHARGE EXCHANGE CROSS SECTION

Let in the energy region  $E > \frac{1}{2} \Delta E$  the solutions (20) determined by the conditions (24) and (25) respectively be of the form

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\* Since the functions  $a(R)$  and  $b(R)$  vanish exponentially inside the potential barriers for  $R=0$  the conditions (33), (34) may be given for numerical integration without essential error for some small  $R_0 \neq 0$ . In our paper  $R_0 = 0.2$ . This corresponds to the replacement of potentials  $Eg + \frac{1}{2M_{gg}} H_{gg}$  and  $Eh + \frac{1}{2M_{hh}} H_{hh}$  by the infinite wall at  $R = R_0$ .

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$$I. \begin{cases} a_1^{(1)} = a_0^{(1)} \sin(k_1 R \frac{\pi L}{2} + \delta_1) \\ b_1^{(1)} = b_0^{(1)} \sin(k_2 R \frac{\pi L}{2} + \delta_2) \end{cases} \quad II. \begin{cases} a_1^{(2)} = a_0^{(2)} \sin(k_1 R \frac{\pi L}{2} + \delta_1) \\ b_1^{(2)} = b_0^{(2)} \sin(k_2 R \frac{\pi L}{2} + \delta_2) \end{cases} \quad (35)$$

where constants  $a_0^{(1)}, b_0^{(1)}$  are determined by numerical integration of (21). According to (32) there is a connection between the coefficients and the phases of (35)

$$k_1 a_0^{(1)} a_0^{(2)} \sin \delta_{21} + k_2 b_0^{(1)} b_0^{(2)} \sin \delta_{12} = 0 \quad (36)$$

where

$$\delta_{21} = \delta_2 - \delta_1; \quad \delta_{12} = \delta_1 - \delta_2 \quad (37)$$

Forming a linear combination from (35) which satisfies the conditions (33), (34) and the normalization for  $R \rightarrow \infty$  we obtain

$$a = \frac{e^{i(k_1 R - \frac{\pi L}{2})} - e^{-i(k_2 R - \frac{\pi L}{2})}}{2ik}; \quad (38)$$

$$b = \frac{k_1 n_1 \sin \delta}{(n_2 e^{-i\delta} - n_1 e^{-i\delta_2 + i\delta}) k_1} \cdot e^{i(k_2 R - \frac{\pi L}{2} + \delta_2)}$$

where

$$n_1 = \frac{b_0^{(1)}}{a_0^{(1)}}, \quad n_2 = \frac{b_0^{(2)}}{a_0^{(2)}}, \quad a = \frac{n_2 e^{i\delta} - n_1 e^{i\delta_2 + i\delta}}{n_2 e^{-i\delta} - n_1 e^{-i\delta_2}}; \quad (39)$$

In accordance with the general theory of inelastic collisions  
charge cross section corresponding to the partial wave  $L$  :

the effective charge ex-

$$\sigma_L \propto \frac{q^2}{E^2} (2L+1) (1-141)^2 \quad (40)$$

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$$\frac{N_1^2 N_2^2 \sin^2 \delta_{21}}{N_1^2 + N_2^2 - 2 N_1 N_2 \cos(\delta_{12} - \delta_{21})} \cdot \frac{k_2}{k_1^3} \quad (40)$$

assuming cross section:

$$\tilde{\sigma}_c = \frac{\pi}{k^2} (2L+1) / L \cdot 2/L = 4\pi (2L+1).$$

$$N_1^2 + N_2^2 - 2 N_1 N_2 \cos(\delta_{12} - \delta_{21})$$

Since the collisions occur at a very low energy the scattering in the S state is the most essential information, if  $\alpha \ll 1$ , the solutions (35) for  $L = 0$  take the form:

$$\begin{aligned} I & \left\{ \begin{aligned} \psi''(R) &= C_1 R + C_2 \\ \psi''(0) &= C_1 R + C_2 \end{aligned} \right. \\ II & \left\{ \begin{aligned} a_0^{(2)} &= C_1 R + C_2 \\ b_0^{(2)} &= C_3 \sin(N_2 R + \delta_2^{(2)}) \end{aligned} \right. \end{aligned} \quad (42)$$

$$k_2^0 = \sqrt{2M\alpha E}$$

and the condition (36) gives

$$C_1^{(2)} C_2^{(2)} - C_2^{(2)} C_1^{(2)} + k_2^0 C_3^{(2)} C_3^{(2)} \sin(\delta_2^{(2)} - \delta_1^{(2)}) = 0$$

The linear combination (38) which satisfies the condition (42) and accordingly normalized has the form:

$$a = 2 \cdot \frac{4G \cdot \tau_{12} e^{i\delta_{21}}}{\delta_2 - \delta_1, e^{i\delta_{21}}} \cdot \frac{\delta_{21} - \frac{\pi}{L} \tau_2 \sin \delta_{21}}{\tau_2 - \tau_1, e^{i\delta_{21}}} \cdot e^{i(\delta_2^{(2)} + \delta_1^{(2)})} \quad (43)$$

where

$$\rho_1 = \frac{C_2^{(2)}}{C_1^{(2)}}, \quad \tau_i = \frac{C_3^{(2)}}{C_i^{(2)}}, \quad \delta_{21} = \delta_2 - \delta_1$$

The charge exchange cross section is:

$$\tilde{\sigma}_{ex} = 4\pi \frac{k_2^0}{k_1} \cdot \frac{\tau_1^2 \tau_2^2 \sin^2 \delta_{21}}{\tau_1^2 + \tau_2^2 - 2\tau_1 \tau_2 \cos \delta_{21}} = 4\pi f \alpha_p^2 \frac{v}{v} \quad (44)$$

and the elastic scattering cross section is

$$\tilde{\sigma}_e = 4\pi \frac{\rho_1^2 \tau_1^2 + \rho_2^2 \tau_2^2 - 2\tau_1 \tau_2 \rho_1 \rho_2 \cos \delta_{21}}{\tau_1^2 + \tau_2^2 - 2\tau_1 \tau_2 \cos \delta_{21}} \alpha_p^2 \quad (45)$$

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If  $V_1$  is the relative velocity of particles,  $N_2$  - the number of nuclei of the isotope with mass  $M_2$  then the probability of the charge exchange is

$$W = N_2 \sigma_{ex} \quad V_1 = 4\pi V_2^0 \frac{\delta_1^2 \delta_2^2 \sin^2 \delta_{21}}{\delta_1^2 + \delta_2^2 - 2\delta_1 \delta_2 \cos \delta_{21}} \quad a_\mu^2 N_2 \quad (46)$$

$$V_2^0 = \sqrt{\frac{4\pi E}{M}}; \quad a_\mu = \frac{\hbar^2}{m_p e^2}$$

The values of  $V_1 \sigma_{ex}$ ,  $f$  and  $\delta_{21}$  for the systems of proton-deuteron, system proton-tritium and deuteron-tritium are given in Table 1.

### SCATTERING OF HEAVY MESIC ATOMS BY LIGHT ISOTOPE NUCLEI,

#### CROSS-SECTIONS AND SCATTERING FUNCTIONS

In the energy region  $-\frac{1}{2} \Delta E \leq E' \leq \frac{1}{2} \Delta E$  when the concentration of the heavy hydrogen isotope is small the most essential process is an elastic scattering of heavy mesic atoms by light isotope nuclei and further after mesic atom slowing down -formation of mesic molecules.

Let the solutions obtained by numerical integration of (21) with boundary conditions (33) and (34) respectively for  $R \rightarrow \infty$  take the form:

$$\begin{aligned} a_b^{(1)}(R) &\approx d_{b_1}^{(1)} e^{-\lambda_1 R} + d_{b_2}^{(1)} e^{\lambda_1 R}; \quad a_s^{(2)}(R) \approx d_{s_1}^{(2)} e^{-\lambda_1 R} + d_{s_2}^{(2)} e^{\lambda_1 R} \\ b_b^{(1)}(R) &\approx d_{b_3}^{(1)} \sin(\lambda_2 R - \frac{\pi}{2} + \omega^{(1)}); \quad b_s^{(2)}(R) \approx d_{s_3}^{(2)} \sin(\lambda_2 R - \frac{\pi}{2} + \omega^{(2)}) \end{aligned} \quad (47)$$

Forming the linear combination from (47) satisfying the condition (26) we obtain within the accuracy of constant factor

$$b_s(R) \approx \sin(\lambda_2 R - \frac{\pi}{2} + \omega) \quad a_s(R) \approx \frac{d_{s_1}^{(1)} d_{s_2}^{(2)} - d_{s_1}^{(2)} d_{s_2}^{(1)}}{\tau} e^{-\lambda_1 R} \quad (48)$$

where

$$\tan \omega = \frac{d_{s_1}^{(1)} d_{s_2}^{(2)} \sin \omega^{(1)} - d_{s_1}^{(2)} d_{s_2}^{(1)} \sin \omega^{(2)}}{d_{s_1}^{(1)} d_{s_2}^{(2)} \cos \omega^{(1)} - d_{s_1}^{(2)} d_{s_2}^{(1)} \cos \omega^{(2)}}; \quad (49)$$

Partial cross section corresponding to the  $s$  wave is

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$$\frac{d\sigma}{dt} (2L+1) \sin^2 \omega = \frac{\pi r}{\lambda^2} (2L+1) \frac{[d_{L_2}^{(N)} d_{L_2}^{(R)} \sin \omega^{(R)} - d_{L_2}^{(R)} d_{L_2}^{(N)} \sin \omega^{(N)}]^2}{T^2} \quad (50)$$

It is very important to know wave functions and effective scattering cross section when the kinetic energy of nucleon atom is small ( $E_N \ll 1$ ). In the region  $R_0 \ll R \ll \frac{1}{\lambda}$ , the solutions (49) for the wave may be represented in the form:

$$a_n^{(N)} \approx d_{L_2}^{(N)} e^{-i\omega^{(N)} t} + d_{L_2}^{(R)} e^{i\omega^{(R)} t} \\ d_{L_2}^{(N)} \approx R + D^{(N)} \quad \omega^{(N)} = \sqrt{2M_N E} \quad (51)$$

The linear combination (51) satisfying the condition (26) in the region  $R_0 \ll R \ll \frac{1}{\lambda}$  is of the form

$$a_n \approx \frac{d_{L_2}^{(N)} - d_{L_2}^{(R)}}{2(R + D^{(N)})} e^{-i\omega^{(N)} t} \quad D \approx R \cdot \frac{d_{L_2}^{(N)} \omega^{(N)} - D^{(N)} d_{L_2}^{(R)}}{d_{L_2}^{(N)} - d_{L_2}^{(R)}}; \quad (52)$$

The normalization of the function (52) is chosen so that for  $R \rightarrow \infty$  it corresponds to a plane wave of the relative motion of nuclei (with coefficient 1) and to a wave that is near a nuclear minimum in Table V the wave functions  $\alpha(R)$  and  $\beta(R)$  for the systems proton-proton, proton-neutron and deuteron-neutron are given. The effective cross section of scattering of a proton with a baryon isotropic by nucleon of lighter mass at small kinetic energy of nucleon atom is

$$\sigma = 4\pi / \left| \frac{d_{L_2}^{(N)} \omega^{(N)} - D^{(N)} d_{L_2}^{(R)}}{d_{L_2}^{(N)} - d_{L_2}^{(R)}} \right|^2 = 4\pi \lambda^2 a_\mu^2 \quad (53)$$

The values  $a_\mu$ ,  $\omega^{(N)}$  and  $\lambda$  are given in Table II. Note that in the case of the process  $p + p \rightarrow p + p$  elastic is not anomalously small as it is indicated in 1/V, since the potential  $d_{L_2}$  which was produced as a result of the charge exchange may be increased in range of the function.

$$E = \frac{1}{2M_N} \ln \frac{E_1}{E_2} \quad (54)$$

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where  $N = \frac{2M_\mu}{N_e + M_\mu} = \frac{4}{9}$  is the mean value of the energy transmitted in  $d_\mu + p$  collision.  $N = 4 \cdot 10^{22}$ ,  $E = 45$  ev is the energy acquired by the mesic atom  $d_\mu$  while charge exchanging.  $E_2 \sim 2 \cdot 10^{-4}$  ev is the final energy. (In this case we make the rough approximation that  $d_\mu$  is believed to move along a straight line since the deviation of  $d_\mu$  in collision with proton can not exceed  $30^\circ$  in the laboratory system of coordinates). The value of the free path according to (54) is  $\ell \sim 0.1$  mm. (The free path owing to the diffusion of  $d_\mu$  with the thermal energy is also of order of 0.1 mm) which is applying less than the experimental 'hole' of about 1 mm.

### MESIC MOLECULE ENERGY LEVELS

Let in the energy region  $E' < -\frac{1}{2} \Delta E$  the solutions of Eq. (21) under the initial conditions (33) and (34) correspondingly have for  $R \rightarrow \infty$  the following form:

$$\begin{aligned} a_i(R) &\approx g_i^{(1)}(E') e^{-x_i R} + g_i^{(2)}(E') e^{x_i R} \\ b_i(R) &\approx g_i^{(1)}(E') e^{-x_2 R} + g_i^{(2)}(E') e^{x_2 R} \end{aligned} \quad (55)$$

Then, forming the linear combination the increasing exponents can be excluded only under the condition:

$$\begin{vmatrix} g_2^{(1)}(E') & g_2^{(2)}(E') \\ g_v^{(1)}(E') & g_v^{(2)}(E') \end{vmatrix} = 0 \quad (56)$$

The condition (56) determines mesic molecule energy levels. By means of the numerical integration of the system (21) for various  $E'$  we can choose the values of  $E'$  satisfying the condition (56). The mesic molecule energy levels obtained in such a way are given in Table III. In Fig. 1 the values of the functions  $a(R)$  and  $b(R)$  are given for the bound state of the mesic molecule ( $p\bar{\nu}_\mu$ ) normalized by the condition

$$\int_0^{\infty} (|a(R)|^2 + |b(R)|^2) dR = 1 \quad (57)$$

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### ENERGY LEVELS OF MESIC MOLECULES WITH IDENTICAL NUCLEI

As we have already pointed out the system for the mesic molecules with identical nuclei breaks up two independent equations. The effective potentials of interaction with corrections taking into account the motion of nuclei\*  $E'g + \frac{1}{2M_{12}} K'gg$  are approximated with a good accuracy by the well-known Morse function:

$$U = A \left[ e^{-2\alpha(R-R_0)} - 2e^{-\alpha(R-R_0)} \right] \quad (58)$$

The values of the effective potentials of interaction are given in /13/. The deviations from the Morse function from the true values of effective potentials of interaction can not change considerably the value of the levels since these deviations are appreciable only in those regions (for very large or very small  $R$ ) where the wave functions decrease exponentially. The values of the energy levels are given in Table III.

### SCATTERING OF MESIC ATOMS BY NUCLEI IDENTICAL TO THE MESIC ATOM NUCLEUS

For small energy of the relative motion of ( $N'R \ll 1$ ) it is not difficult to calculate effective cross section of mesic atom scattering by nuclei of the isotope\*\*. For  $E' > 0$  the solution of the Schrodinger equation with potential (58) is of the form

$$g \cdot e^{-\frac{\alpha R}{2}} i \left[ g^{\frac{i\alpha}{2}} e^{i\alpha g} \left( -\frac{\sqrt{2M_1}}{\alpha} + \frac{1}{2} + i\alpha; 1+2i\alpha; g \right) - g^{\frac{i\alpha}{2}} e^{i\alpha g} \left( -\frac{\sqrt{2M_1}}{\alpha} + \frac{1}{2} - i\alpha; 1-2i\alpha; g \right) \right] \quad (59)$$

\* Compare with /10/, where the approximation was performed without taking into account the corrections to the motion of nuclei.

\*\* The energy of the relative motion must be considerably higher than the energy of the superfine splitting in the mesic atom (in the case of the mesic atom  $P_1$  it is about of 0.8 ev). The effects at lower energy are considered in /5/.

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where

$$\beta = \frac{2\sqrt{2M\Lambda}}{\alpha} e^{i\theta}; \quad x = \frac{\sqrt{2ME'}}{\alpha} = \frac{\kappa}{\alpha}; \quad e^{i\varphi} = \frac{\Gamma(1+2i\varphi)\Gamma(-\frac{\sqrt{2M\Lambda}}{\alpha} + \frac{1}{2} - i\varphi)}{\Gamma(1-2i\varphi)\Gamma(-\frac{\sqrt{2M\Lambda}}{\alpha} + \frac{1}{2} + i\varphi)}$$

For  $R \rightarrow \infty (\beta \rightarrow 0)$  this solution equals asymptotically:

$$g \sim \sin \left( kR - \frac{\kappa R n 2\sqrt{2M\Lambda}}{\alpha} - kR_0 + \varphi \right) \quad (60)$$

If the energy of the relative motion is rather small ( $kR_0 \ll 1$ ) then, in the region  $R \ll R \ll \frac{1}{k}$

$$g \sim R - \lambda g \quad (61)$$

The scattering length may be obtained from (64), (65)

$$\lambda_g = \left\{ \Psi \left( -\frac{\sqrt{2M\Lambda}}{\alpha} + \frac{1}{2} \right) - 2\psi(1) + \ln \frac{2\sqrt{2M\Lambda}}{\alpha} \right\} \frac{\alpha}{2M\Lambda} + R_0 \quad (62)$$

where  $\Psi(x) = \frac{\Gamma'(x)}{\Gamma(x)}$ . Note that if the value  $\left( \frac{\sqrt{2M\Lambda}}{\alpha} - \frac{1}{2} \right)$  is close to the integer number this corresponds to the fact that the nucleic molecule has a level (real or virtual with the energy close to zero) then, due to the fact that the function  $\Psi(x)$  has poles for integer negative numbers  $x$  the value of  $\lambda_g$  may be very large. The possibility of such a resonance has been observed in [2].

The solution of the second equation (21) may be also obtained easily if the potential  $V_u = E'_u(R) + \frac{1}{2M_u} K'_{uu}(R)$  is approximated by an exponent:

$$V_u = Be^{-\beta R} \quad (63)$$

this may be made with a good accuracy for the values of  $R$  essential for the scattering. The Schrödinger equation with potential (63) reduces to the Bessel equation of the imaginary argument by introducing a new variable quantity. Thus, for  $E' = 0$

$$h(R) = \frac{2}{\beta} K_0 \left( \frac{2\sqrt{2M\Lambda}}{\beta} e^{-\beta R/2} \right)$$

where  $K_0(i)$  is the Hankel function of the imaginary argument. The normalization of the function  $M_u$  is chosen so that for  $R \rightarrow \infty$  the solution will be of the form:

$$h(R) \approx R - \lambda u \quad (64)$$

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(taking into account that for  $t \rightarrow 0, K_0(t) = \ln^2 t - C$  where  $C = 0.577$  is the Euler constant) we obtain

$$\lambda_u = \frac{2}{\beta} \left[ C + \ln \frac{\sqrt{2MB}}{\beta} \right]. \quad (65)$$

Taking into account that the mesonic function  $\Sigma_g(\vec{r}, R)$  is symmetrical with respect to the nucleon exchange and  $\Sigma_u(\vec{r}, R)$  is antisymmetrical, it may be concluded that in the S-wave the relative motion of identical nuclei will be described by the function  $g(R)$ , if the summary spin of nuclei is even and by the function  $h(R)$ , if the summary spin of nuclei is odd. The effective cross section of the hydrogen mesic atom scattering by protons (and mesic atoms of tritium by tritium nuclei) at small energies is to be of the form:

$$\sigma = 2\pi \left( \frac{1}{4} \frac{\lambda_g^2}{1 + \kappa^2 \lambda_g^2} + \frac{3}{4} \frac{\lambda_u^2}{1 + \kappa^2 \lambda_u^2} \right) \quad (66)$$

$$\sigma = 2\pi \left( \frac{2}{3} \frac{\lambda_g^2}{1 + \kappa^2 \lambda_g^2} + \frac{1}{3} \frac{\lambda_u^2}{1 + \kappa^2 \lambda_u^2} \right). \quad (67)$$

The formula (66) is analogous to that describing the scattering of neutrons by protons. For large  $\lambda_g$  in accordance with (66) there will be a resonance in the scattering when  $\kappa \rightarrow 0$ . (The terms  $\lambda_g$  and  $\lambda_u$  in (66) are written for better analogy with the well-known deuteron formula, the consideration of these terms represents essentially the exceeding of the accuracy since  $\lambda_u$  corresponds to the repulsion and must coincide in magnitude with the radius of action of forces:  $\lambda_u = \lambda_p$ ). This takes place just in the scattering  $p_n + p \rightarrow p_d + p$  since the mesic atoms ( $pp$ ) have a virtual level with the energy close to 0. The effective scattering cross section  $\sigma_{pd} + \sigma_{pp}$  calculated according to (66) coincides well with the cross section given in [7]. In the scattering  $d_n + d$  the cross section calculated according to (67) turns out to be twice as large as given in [7].

#### MESIC MOLECULE FORMATION

Moving in the matter the hydrogen mesic atoms in virtue of their neutrality can go easily through the electronic shells of the hydrogen molecules and approaching the nuclei they can form mesic molecules (more exactly, mesic molecular ions)  $(pp)_m^+$ ,  $(pd)_m^+$ , etc. (like the well-known molecular ions  $H_2^+$ ,  $HHD^+$ , etc.). The binding energy of the mesic molecule, in principle, may be transmitted to the radiation, electron shell and to the nucleus coupled by

chemical forces with that which forms the mesic molecule.

The last transition, however, may have some meaning only in the case when the mesic molecule forms itself in the state with very small coupling energy (of order of the coupling energy of the ordinary molecule). As we can see from Table III this condition can not be fulfilled for any of the molecules (perhaps, except for (dd)). Since the size of the mesic molecule is less than the atomic one the relations between the probabilities of formation of a mesic molecule in a radiation way and by means of the recoil of the energy to the electron of the shell may be expressed using the standard theory of intrinsic conversion of electrons under nuclear transitions. In the considered region of transmitted energies (tens - hundreds of ev) the coefficients of intrinsic conversion are very large; therefore the probability of the radiative formation of mesic molecules is less incomparably than the conventional one. Because the formation of mesic molecules takes place at small relative energies of mesic atoms an electric dipole transition from the S-wave of continuous spectrum with the conversion on electron will be the most important one. Let  $P_1$ ,  $P_2$ ,  $P_M$  be distances from the nuclei  $M_1$ ,  $M_2$  and  $M$  meson respectively from an arbitrary point; then the Coulomb field of the system at large distances has the form

$$\frac{e}{P} + \frac{e}{P_2} - \frac{e}{P_M} \approx \frac{e}{P} + \frac{dp}{P^3} \quad (68)$$

where  $P$  is the distance from the center of mass of nuclei, and  $d$  is the dipole momentum of the system

$$\vec{d} = \frac{e}{2} \frac{M_2 - M_1}{M_2 + M_1} \vec{R} - \frac{e}{2} (\vec{r}_1 + \vec{r}_2) \\ (\vec{R} = \vec{r}_2 - \vec{r}_1; \vec{r}_1 = \vec{r} - \vec{r}_1; \vec{r}_2 = \vec{r} - \vec{R}_2). \quad (69)$$

If the precise Coulomb functions of the hydrogen atom are taken as wave functions of electron then in analogy with the probability of conversion under the nuclear transition we obtain for the probability of mesic molecule formation by means of dipole conversion:

$$W = \frac{16\pi^2}{3} \frac{e^2}{v^2} \cdot \frac{2\pi^2 e^{-4\pi v^2/3}}{(1+e^{-2\pi v^2})} \frac{N}{M_1} \sum | \langle d \rangle |^2. \quad (70)$$

$$\text{where } Q = \frac{e^2}{4\pi\varepsilon_0} \quad q = \frac{e^2}{4\pi\varepsilon_0}$$

( $v$ ,  $\nu$  - mean end velocity),  $N$  is the number of nuclei in  $\text{cm}^3$ ,  $\langle d \rangle$  is the matrix element of the dipole interaction between the wave functions of the mesic molecule related to the

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and the discrete spectrum respectively.

$$\langle d \rangle = \int \psi_{\text{mol}}^* d\psi_{\text{mol}} (dR)(dz_\mu)$$

The integration in (71) is performed over the coordinates of  $\mu$ -meson and of nuclei. The integration over the nuclear coordinates in (71) leads to the integral:

$$d_R \cdot \int \Sigma_g \frac{1}{2} (r_1 + r_2) \Sigma_u (dz_\mu) \quad (72)$$

From the nature of symmetry it is clear that  $d_R$  is directed along  $R$ :  $d_R = f(R)$ , where  $f(R)$  is a function of  $R$ . In the approximation (LCAO)

$$f(R) = \frac{1}{2} \frac{1}{\sqrt{1-e^{-2R}} (1+R)^2} \approx -\frac{1}{2}, \text{ for } R > 3 \quad (73)$$

The approximation LCAO fails for small  $R$ , however, it is clear that since the function (73) corresponding to the repulsion state  $\Sigma_u$  for small values of  $R$  vanishes exponentially the small values of  $R$  contributes insignificantly to the matrix element  $\langle d \rangle$ . Note that the normalization  $\Sigma_g = \Sigma_u$  is well fulfilled for small  $R$  too, if the calculation of  $d_R$  is made with the positive functions  $\Sigma_g$ ,  $\Sigma_u$ . The summation in (70) is made over all possible final states of the nuclei molecule. In calculating the normalization of the wave function of the continuous spectrum we assumed to be chosen so that at the infinity there a plane wave with the coefficient one, and the wave function of the discrete spectrum is, normalized to unity. In this case the matrix element  $\langle d \rangle$  will have the dimension of  $(\alpha_e)^2 (\omega_e \cdot \hbar^2 / m_e e^2)$ . Separating out dimension factor it is convenient to rewrite the formula (70) in the form

$$W = \frac{16}{3} \left( \frac{\alpha_e^2}{c} \right) \left( \frac{m_e}{m_{\text{mol}}} \right)^2 \frac{e^2 2\pi^2 e^{-4\pi c/\hbar}}{3\alpha_e (1+\rho^2)(1-e^{-2R})} \left\{ \sum \left| \langle d \rangle \right|^2 \right\}, \quad (74)$$

where the matrix element  $\langle d \rangle$  is calculated in dimensionless ("nuclear units") units. It is easy to see that the first term in the dipole momentum (69) has non-zero matrix elements only for transitions with mesonic functions of identical parity  $\Sigma_g \rightarrow \Sigma_g$ ;  $\Sigma_u \rightarrow \Sigma_u$  while the second term gives transitions only between mesonic functions of different parity ( $\Sigma_u \rightarrow \Sigma_g$ ;  $\Sigma_g \rightarrow \Sigma_u$ ). The dipole transitions from the S-wave of the continuous spectrum may occur only into the rotation state with  $L = 1$ . From the table III one can see that for all mesic molecules (except  $^{11}_N$ ) the transition is performed into the ground rotation state ( $L = 1$ ;  $\lambda = 0$ ). For mesic molecules  $^{11}_N$  the transition into the vibration-rotational state is possible too.

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The probability of formation of mesic molecules with identical nuclei  $\ell = 1; n = 1$  may be calculated analogously as it was made for mesic molecules in (dd). The probabilities of formation of these molecules are given in the Table 4. In the case of mesic molecules with different nuclei of greatest interest is the calculation of probability of mesic molecule formation when the collision of the mesic atom of heavier isotope with lighter nucleus occurs, for example  $d_{\alpha} + p \rightarrow (dp)_{\mu}$  (in the collision  $D_{\mu} + d$  the charge exchange is the most probable process  $p_{\mu} + d \rightarrow d_{\mu} + p$ ). The wave function of the initial state has the form:

$$\psi_{\text{init}}^{(1)} = C_0(R) \Sigma_y + H_0(R) \Sigma_u, \quad (73)$$

where the functions  $C_0, H_0$  are connected with the functions  $G_0(R), L_0(R)$  by the equations (16) and the wave function of the final state corresponding to the rotation level of the mesic atom with quantum number  $\ell$  has the form

$$\psi_{\text{final}}^{(1)} = \left\{ G_0(R) + L_0(R) \Sigma_u \right\} \cdot \frac{1}{2} Y_{1,1}(0,4). \quad (74)$$

$$\int \psi_{\text{final}}^{(1)} \psi_{\text{init}}^{(1)} dR = \int \left[ G_0(R) + L_0(R) \Sigma_u \right] \left[ G_0(R) + L_0(R) \Sigma_u \right] dR. \quad (75)$$

Integration of (75) can be performed numerically. The probability of formation of mesic molecules

with different nuclei  $\ell = 1; n = 1$  in accordance with (77) and (76) are given in the

## APPENDIX

The functions  $\Sigma_y$  and  $\Sigma_u$  (1) are calculated to 1/3 by means of precise wave functions (16) and (17). We give for comparison the values  $N_{yy}(R)$  and  $N_{uu}(R)$  in LCRD consistent for large  $R$ .

$$\begin{aligned} \Sigma_y &= \left( 2S(n-s) \right)^{1/2} \left( e^{-r_1} + e^{-r_2} \right) \\ \Sigma_u &= \left( 2S(n-s) \right)^{1/2} \left( e^{-r_1} - e^{-r_2} \right) \quad (\text{LCRD}) \\ S &= \left( 1 + 2 + \frac{4}{3} \right) e^{-2} \end{aligned}$$

D.1.

D.2.

The expression (16,1) consistent for small  $R$ , when  $\Sigma_y$  and  $\Sigma_u$  transform re-

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respectively the 1s and 2p levels of He.

$$\Sigma_g = \frac{1}{N_g} e^{-(r_1 + r_2)} \\ \Sigma_u = -\frac{1}{N_u} e^{-i\theta_2(r_1 + r_2)} (r_1 \cos \theta_1, -r_2 \cos \theta_2)$$

D.3

$$N_g^2 = \frac{3}{8} e^{-2R} (1 + 2R + \frac{5}{4} R^2)$$

D.4

$$N_u^2 = 45 \left( 1 + R + \frac{9}{20} R^2 + \frac{7}{60} R^3 + \frac{1}{60} R^4 \right) e^{-2R}$$

D.4'

In the approximation (LCAO):

$$K_{gg} = \frac{1}{2} - \frac{5}{2(1+3)} - \frac{1}{36} \frac{R^2(1+R)^2}{(1-3)^2} e^{-2R}$$

D.5

$$K_{uu} = \frac{1}{2} + \frac{5}{2(1-3)} - \frac{1}{36} \frac{R^2(1+R)^2}{(1-3)^2} e^{-2R}$$

D.5'

In the approximation (U.A):

$$K_{gg} = 1 - \left( \frac{N_g'}{N_g} \right)^2 \approx_{R \rightarrow 0} 1 - \frac{16}{9} R^2$$

D.6

$$K_{uu} = \frac{2}{R^2} + \frac{1}{4} - \left( \frac{N_u'}{N_u} \right)^2 \approx_{R \rightarrow 0} \frac{2}{R^2} + \frac{1}{4}$$

D.6'

For comparison in Fig. 2,3 are given the values  $K_{gg}$  and  $K_{uu}$ , calculated in [15] and according to the approximation (LCAO) and (U.A). For the value  $Q(R)$  we get:

$$(LC10) Q = \frac{M_a - M_i}{M_a + M_i} \cdot \frac{R(R+1)e^{-2R}}{6\sqrt{1-3^2}}$$

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$$(U1)Q = -\frac{M_2 - M_1}{M_2 + M_1} \cdot \frac{16\pi}{81} \cdot \frac{e^{-\frac{5R}{2}}}{N_g N_u} \left( 1 + \frac{3}{2}R + \frac{3}{4}R^2 \right).$$

We note that  $Q$  calculated in accordance with (LCAO) for  $R \rightarrow 0$  coincide well with the values of  $Q$ , calculated in accordance with

$$(LCAO) \begin{cases} K_{gu} = \frac{1}{2\sqrt{1-s}} \left\{ 1 + e^{-R} \left( 1 + R - \frac{R^2}{3} \right) - \frac{R^2(1+R)^2}{9(1-s)} e^{-2R} \right\} \frac{M_2 - M_1}{M_2 + M_1}, \\ K_{ug} = \frac{1}{2\sqrt{1-s^2}} \left\{ 1 - e^{-R} \left( 1 + R - \frac{R^2}{3} \right) - \frac{R^2(1+R)^2}{9(1+s)} e^{-2R} \right\} \frac{M_2 - M_1}{M_2 + M_1}. \end{cases}$$

Thus, in the all region  $R$  the approximation  $Q = Q_{LCAO}$  may be taken.

The quantities  $K_{gu}(R)$  and  $K_{ug}(R)$  in the approximation (LCAO) and (U A) equal cor-

$$(U A) \begin{cases} K_{gu} = \frac{M_2 - M_1}{M_2 + M_1} \cdot \frac{64\pi}{81} \cdot \frac{e^{-\frac{5R}{2}}}{N_g N_u} \left( 1 + \frac{3}{2}R + \frac{3}{4}R^2 \right) \left( 1 - \frac{R}{2} \frac{N_u}{N_g} \right) \frac{1}{R}, \\ K_{ug} = \frac{M_2 - M_1}{M_2 + M_1} \cdot \frac{64\pi}{81} \cdot \frac{e^{-\frac{5R}{2}}}{N_g N_u} \left\{ \frac{1}{2} \frac{N_g}{N_u} \left( 1 + \frac{3}{2}R + \frac{3}{4}R^2 \right) + \right. \\ \left. + \frac{3}{8}R + \frac{9}{16}R^2 \right\}. \end{cases}$$

Show the behaviour of the function for  $R \rightarrow 0$ .

$$K_{gu} (LCAO) \rightarrow \frac{M_2 - M_1}{M_2 + M_1} \cdot \frac{2}{\sqrt{3}R} \approx 1.15 \frac{1}{R} \frac{M_2 - M_1}{M_2 + M_1};$$

$$K_{gu} (U A) \rightarrow \frac{M_2 - M_1}{M_2 + M_1} \cdot \frac{64\sqrt{2}}{81R} \approx 1.12 \frac{1}{R} \frac{M_2 - M_1}{M_2 + M_1},$$

$$K_{ug} (LCAO) \rightarrow \frac{M_2 - M_1}{M_2 + M_1} \cdot \frac{7\sqrt{2}}{18} R \approx 0.67 R \frac{M_2 - M_1}{M_2 + M_1},$$

$$K_{ug} (U A) \rightarrow \frac{M_2 - M_1}{M_2 + M_1} \cdot \frac{8\sqrt{2}}{35} R \approx 0.322 \frac{M_2 - M_1}{M_2 + M_1},$$

Table I  
charge exchange cross section

	$p_\mu + d \rightarrow d_\mu + p$	$p_\mu + t \rightarrow t_\mu + p$	$d_\mu + t \rightarrow t_\mu + d$
$f$	2.1	0.84	0.0067
$\sigma_{\mu} \cdot v$	$3.42 \cdot 10^{-13} \frac{\text{cm}^2}{\text{sec}}$	$1.49 \cdot 10^{-13} \frac{\text{cm}^2}{\text{sec}}$	$1.15 \cdot 10^{-15} \frac{\text{cm}^3}{\text{sec}}$
$\sigma_\mu$	$1.98 \cdot 10^{-19} \text{ cm}^2$	$1.53 \cdot 10^{19} \text{ cm}^2$	$2.41 \cdot 10^{-19} \text{ cm}^3$

Table II  
mesic atom elastic scattering cross section

	$d_\mu + p \rightarrow d_\mu + p$	$t_\mu + p \rightarrow t_\mu + p$	$t_\mu + d \rightarrow t_\mu + d$
$\lambda$	2.03	2.66*	6.7
$\sigma_{\mu}(0)$	$3.39 \cdot 10^{-20} \text{ cm}^2$	$5.84 \cdot 10^{-20} \text{ cm}^{2**}$	$36.9 \cdot 10^{-20} \text{ cm}^2$

Table III  
mesic molecule levels ( ev ) (for mesic molecules with various nuclei the energy  
levels are calculated from the level of heavier isotope )

	$L=0$		$L=0$		$L=2$	$L=3$
	$n=0$	$n=1$	$n=0$	$n=1$	$n=0$	$n=0$
$(pp)_\mu^+$	252	-	109	-	-	-
$(dd)_\mu^+$	330	40	227	??	88	-
$(uu)_\mu^+$	367	86	283	45	170	50
$(pd)_\mu^+$	220	-	90	-	-	-
$(pt)_\mu^+$	213	-	93	-	-	-
$(dt)_\mu^+$	318	32	232	-	102	-

\* \*\* These values are derived for  $K_{K\bar{K}} = 0$ . Calculations with correct value of  $K_{K\bar{K}}$  give for  $\lambda$  the value  $\sim 10$ , what seems to be doubtful.

Table IV  
probabilities of mesic molecule formation in units  $10^6 \text{ sec}^{-1}$  in liquid hydrogen

$(\text{pp})$	$(\text{dd})$	$(\text{tt})$	$(\text{pd})$	$(\text{dt})$	$(\text{pt})$
1.53	0.006	0.38	0.7	$\sim 0.001$	0.25

In the present paper the probabilities of the mesic molecule formation should be considered to be correct only in order of value, since the binding of hydrogen nuclei in molecules of  $\text{H}_2$  has been neglected in our calculations what leads apparently to the increase of  $W$ .

In the case of the mesic molecular ion (dd) and (dt) 0-0 transitions may contribute to the molecule formation probability, due to the oscillation level close to zero ( $L \approx 0$ ).

We have just received preprints by Cohen, Judd and Riddell. They calculate the probability of the formation of mesic molecules with identical nuclei using the wave functions normalised to  $\sqrt{2}$ , but not to unity, what doubles the value of  $W$ .

We wish to thank Y.B. Zeldovich for initiating this investigation and for many useful discussions.

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Table V

Wave functions of mesic molecular ions (unnormalized) in state L = 1

	$(\rho \sigma)$ Level 90 ev	$(\rho \sigma)$ Level 98 ev	$(\rho \sigma)$ Level 232 ev	R
0,3	$0,114 \cdot 10^{-2}$	$0,110 \cdot 10^{-2}$	$0,116 \cdot 10^{-2}$	0,3
0,5	$0,530 \cdot 10^{-2}$	$0,516 \cdot 10^{-2}$	$0,554 \cdot 10^{-2}$	0,5
0,7	$0,132 \cdot 10^{-1}$	$0,130 \cdot 10^{-1}$	$0,141 \cdot 10^{-1}$	0,7
0,9	$0,296 \cdot 10^{-1}$	$0,294 \cdot 10^{-1}$	$0,277 \cdot 10^{-1}$	0,9
1,1	$0,423 \cdot 10^{-1}$	$0,424 \cdot 10^{-1}$	$0,462 \cdot 10^{-1}$	1,1
1,3	$0,625 \cdot 10^{-1}$	$0,632 \cdot 10^{-1}$	$0,687 \cdot 10^{-1}$	1,3
1,5	$0,849 \cdot 10^{-1}$	$0,869 \cdot 10^{-1}$	$0,936 \cdot 10^{-1}$	1,5
1,7				1,7
1,9	$0,130$	$0,137$	$0,144$	1,9
2,1	$0,191$	$0,160$	$0,165$	2,1
2,3	$0,168$	$0,181$	$0,183$	2,3
2,5	$0,182$	$0,198$	$0,197$	2,5
2,7	$0,191$	$0,211$	$0,206$	2,7
2,9	$0,197$	$0,221$	$0,210$	2,9
3,1	$0,198$	$0,226$	$0,219$	3,1
3,3	$0,196$	$0,228$	$0,203$	3,3
3,5	$0,191$	$0,226$	$0,198$	3,5
3,7	$0,184$	$0,221$	$0,189$	3,7
3,9	$0,179$	$0,215$	$0,177$	3,9
4,1	$0,164$	$0,206$	$0,164$	4,1
4,3	$0,152$	$0,196$	$0,151$	4,3
4,5	$0,140$	$0,184$	$0,137$	4,5
4,7	$0,120$	$0,173$	$0,124$	4,7
4,9	$0,116$	$0,161$	$0,111$	4,9
5,1	$0,104$	$0,149$	$0,989 \cdot 10^{-1}$	5,1
5,3	$0,935 \cdot 10^{-1}$	$0,137$	$0,875 \cdot 10^{-1}$	5,3
5,5	$0,833 \cdot 10^{-1}$	$0,126$	$0,770 \cdot 10^{-1}$	5,5
5,7	$0,738 \cdot 10^{-1}$	$0,115$	$0,673 \cdot 10^{-1}$	5,7
5,9	$0,650 \cdot 10^{-1}$	$0,105$	$0,585 \cdot 10^{-1}$	5,9
6,0	$0,609 \cdot 10^{-1}$	$0,100$	$0,545 \cdot 10^{-1}$	6,0
6,2	$0,534 \cdot 10^{-1}$	$0,908 \cdot 10^{-1}$	$0,471 \cdot 10^{-1}$	6,2
6,4	$0,465 \cdot 10^{-1}$	$0,822 \cdot 10^{-1}$	$0,401 \cdot 10^{-1}$	6,4
6,6	$0,403 \cdot 10^{-1}$	$0,742 \cdot 10^{-1}$	$0,341 \cdot 10^{-1}$	6,6
6,8	$0,348 \cdot 10^{-1}$	$0,669 \cdot 10^{-1}$	$0,290 \cdot 10^{-1}$	6,8
7,0	$0,299 \cdot 10^{-1}$	$0,601 \cdot 10^{-1}$	$0,247 \cdot 10^{-1}$	7,0
7,2	$0,255 \cdot 10^{-1}$	$0,539 \cdot 10^{-1}$	$0,198 \cdot 10^{-1}$	7,2
7,4	$0,216 \cdot 10^{-1}$	$0,483 \cdot 10^{-1}$	$0,158 \cdot 10^{-1}$	7,4
7,6	$0,181 \cdot 10^{-1}$	$0,432 \cdot 10^{-1}$	$0,122 \cdot 10^{-1}$	7,6
7,8	$0,150 \cdot 10^{-1}$	$0,385 \cdot 10^{-1}$	$0,865 \cdot 10^{-1}$	7,8
8,0	$0,127 \cdot 10^{-1}$	$0,342 \cdot 10^{-1}$	$0,557 \cdot 10^{-1}$	8,0
8,2	$0,677 \cdot 10^{-2}$	$0,293 \cdot 10^{-1}$	$0,136 \cdot 10^{-1}$	8,2
			$0,353 \cdot 10^{-1}$	
			$0,112 \cdot 10^{-2}$	
			$0,253 \cdot 10^{-2}$	

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Table V (continued)

Wave functions. Scattering of mesic atom with a heavier isotope by nuclei of lighter one  
at zero energy

	$d_p \cdot p$	$t_p \cdot p$	$t_p \cdot d$	
R	a(R)	b(R)	c(R)	d(R)
0,3	-0,469 $10^{-1}$	-0,454 $10^{-1}$	-0,118 $10^{-1}$	-0,164 $10^{-1}$
0,5	-0,181 $10^{-1}$	-0,178 $10^{-1}$	-0,435 $10^{-1}$	-0,383 $10^{-1}$
0,7	-0,386 $10^{-1}$	-0,382 $10^{-1}$	-0,926 $10^{-1}$	-0,935 $10^{-1}$
0,9	-0,655 $10^{-1}$	-0,655 $10^{-1}$	-0,141	-0,164
1,1	-0,965 $10^{-1}$	-0,974 $10^{-1}$	-0,245	-0,248
1,3	-0,129	-0,131	-0,329	-0,337
1,5	-0,159	-0,160	-0,407	-0,423
1,7	-0,183	-0,190	-0,470	-0,476
1,9	-0,201	-0,210	-0,513	-0,549
2,1	-0,210	-0,221	-0,532	-0,576
2,3	-0,210	-0,221	-0,526	-0,577
2,5	-0,202	-0,213	-0,498	-0,549
2,7	-0,186	-0,195	-0,449	-0,496
2,9	-0,163	-0,168	-0,385	-0,420
3,1	-0,137	-0,135	-0,310	-0,325
3,3	-0,107	-0,948 $10^{-1}$	-0,229	-0,215
3,5	-0,754 $10^{-1}$	-0,508 $10^{-1}$	-0,145	-0,940 $10^{-1}$
3,7	-0,440 $10^{-1}$	-0,378 $10^{-2}$	-0,631 $10^{-1}$	+0,341 $10^{-1}$
3,9	-0,136 $10^{-1}$	+0,491 $10^{-1}$	+0,142 $10^{-1}$	0,166
4,1	+0,148 $10^{-1}$	0,948 $10^{-1}$	0,849 $10^{-1}$	0,299
4,3	0,407 $10^{-1}$	0,149	0,147	0,431
4,5	0,636 $10^{-1}$	0,194	0,201	0,561
4,7	0,832 $10^{-1}$	0,242	0,245	0,687
4,9	0,996 $10^{-1}$	0,289	0,280	0,808
5,1	0,113	0,334	0,306	0,925
5,3	0,123	0,378	0,325	1,04
5,5	0,130	0,420	0,337	1,15
5,7	0,139	0,461	0,342	1,25
5,9	0,138	0,500	0,343	1,35
6,0	0,139	0,519	0,341	1,40
6,2	0,139	0,556	0,335	1,49
6,4	0,137	0,593	0,326	1,58
6,6	0,134	0,628	0,314	1,67
6,8	0,130	0,662	0,300	1,75
7,0	0,126	0,696	0,284	1,84
7,2	0,121	0,729	0,269	1,93
7,4	0,115	0,761	0,253	1,99
7,6	0,110	0,793	0,237	2,08
7,8	0,104	0,824	0,221	2,15
8,0	0,981 $10^{-1}$	0,859	0,205	2,13
8,2	0,844 $10^{-1}$	0,931	0,171	2,42
8,4	0,724 $10^{-1}$	1,01	0,142	2,60
8,6	0,623 $10^{-1}$	1,08	0,116	2,78
10,0	0,545 $10^{-1}$	1,15	0,939 $10^{-1}$	2,95
10,2	0,488 $10^{-1}$	1,22	0,739 $10^{-1}$	3,13
11,0	0,457 $10^{-1}$	1,30	0,598 $10^{-1}$	3,31
11,5	0,453 $10^{-1}$	1,37		
12,0	0,476 $10^{-1}$	1,44		

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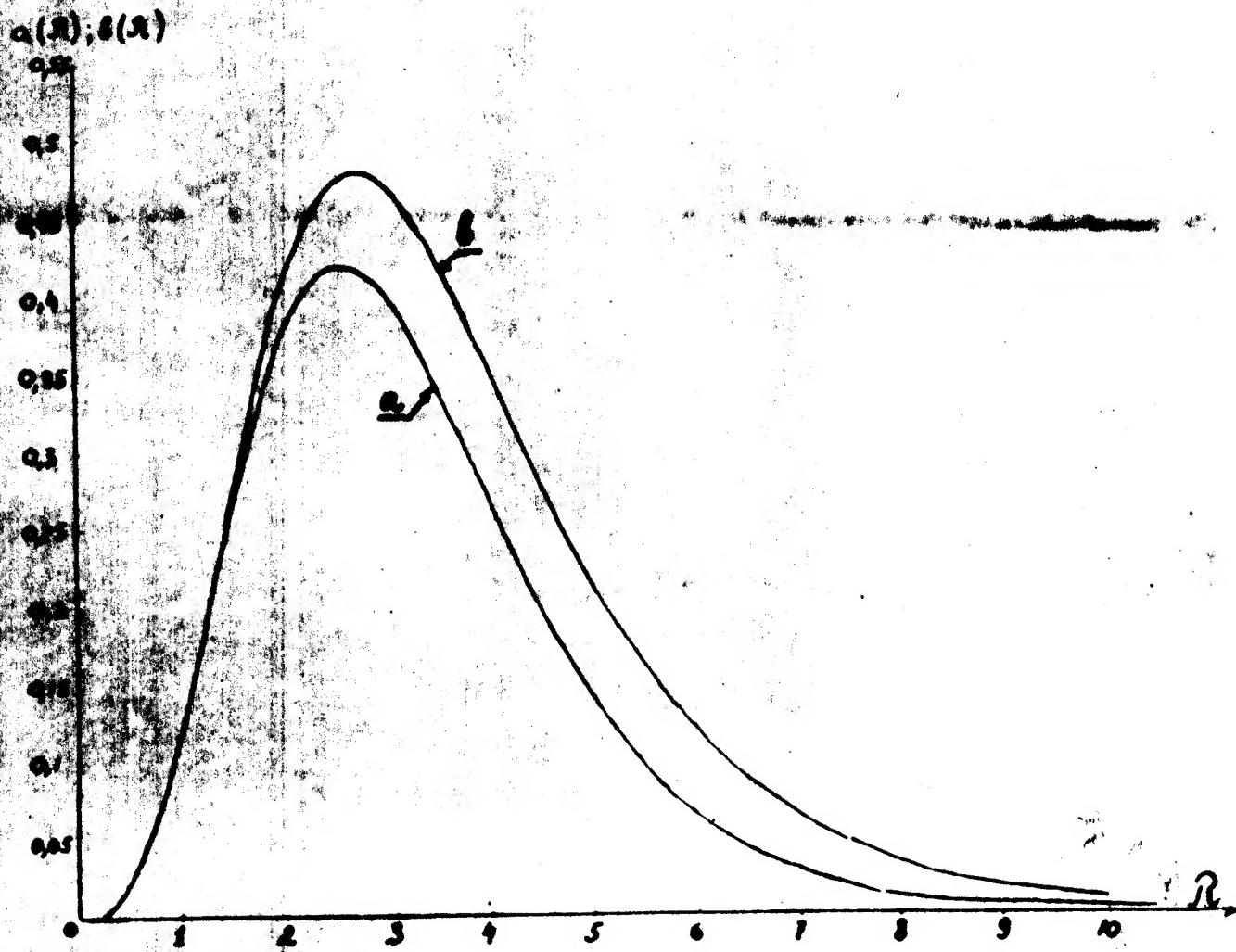


Fig. 1.

Wave functions of basic molecular ion ( $pt_{\infty}$ )<sub>00</sub> corresponding to the energy level 98 ev. The functions are given for illustration of qualitative behaviour of  $\alpha(R)$  and  $\delta(R)$  for discrete values  $E$ . (Exact values of function are given in Table V).

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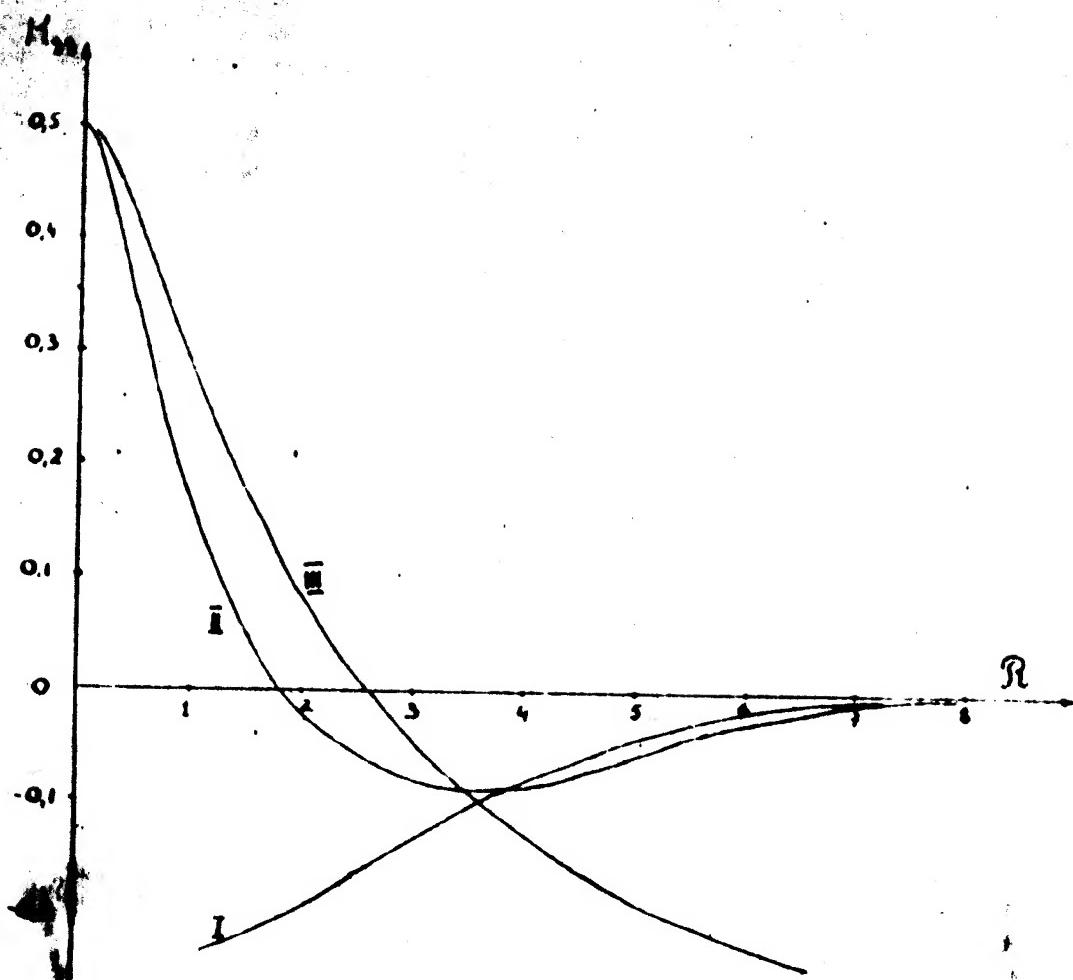


Fig. 2. Function  $K_{gg}$  calculated: I-in (LCAO)-approximation; II - with exact functions; III - in (UA) -approximation.

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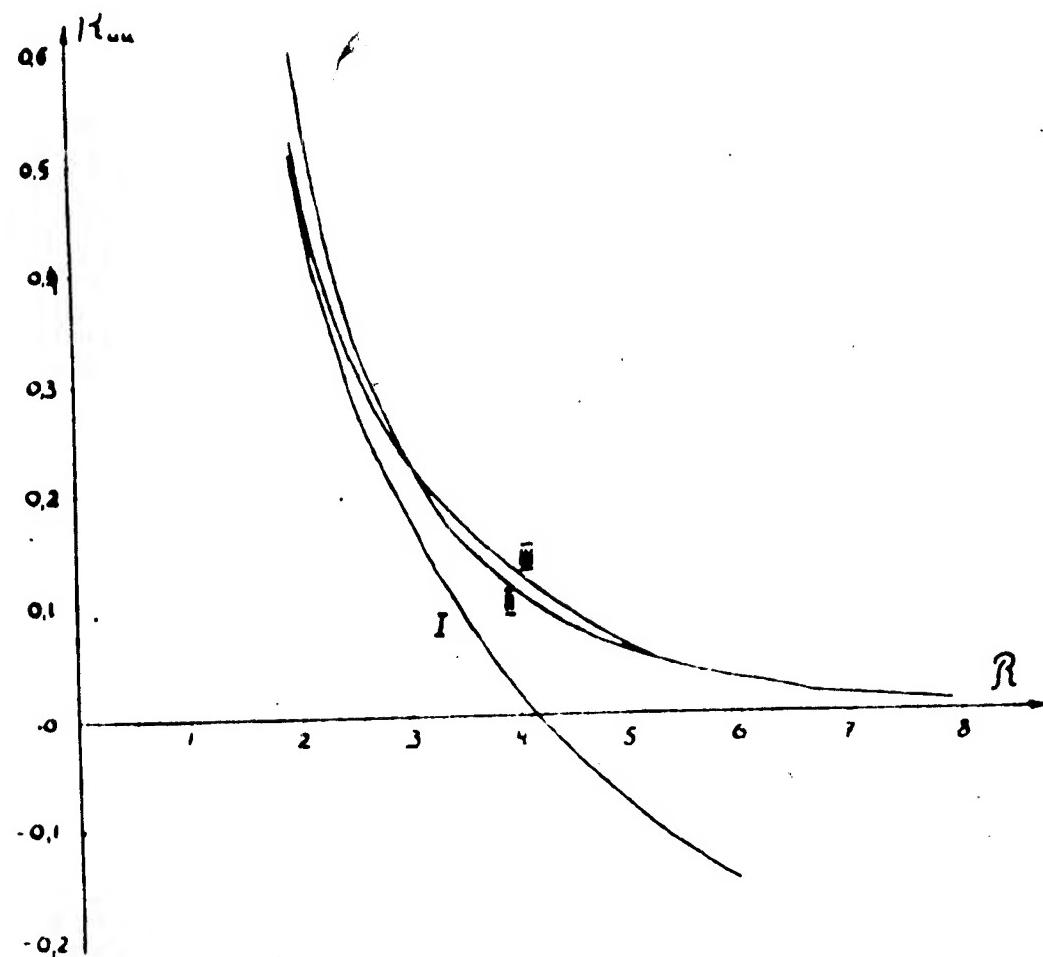


Fig. 3. Function  $K_{uu}$  calculated: I -in (UA)-approximation; II -in (LCAO) approximation;  
III -with exact functions /15/.

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